

# Multi-particle entanglement and generalized $N$ -particle teleportation using quantum statistical correlations

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Construction of multi-particle entangled states and direct teleportation of  $N$ -(spin  $1/2$ ) particles are important areas of quantum information processing. A number of different schemes which have been presented already, address the problem through controlled teleportation. In this article, a criterion based on standard quantum statistical correlations employed in the many-body virial expansion is used to determine maximum entanglement for a  $N$ -particle state. These states remain entangled through proper traces to states for a smaller number of particles and can be generalized for arbitrary number of particles. It is shown that they are quite useful in generalized,  $N$ -particle, direct teleportation. The corresponding quantum gates are also indicated for teleportation schemes from simple computational basis states.

## I. INTRODUCTION

Quantum teleportation deals with the transfer of information from one remote location to another without physical transport of the information or measurement on either side to confirm and verify the information content [1]. It rests on the quantum correlations for which there are no classical analogues. Bennett and coworkers proposed a theoretical scheme in 1993 using the celebrated Bohm-Aharonov-Einstein-Podolsky-Rosen (EPR) pair and demonstrated the transfer of two bits of information without classical communication with a probability of  $1/4$  and with classical communication (with speeds less than that of light in vacuum) with probability  $3/4$  [2,3]. Transfer of the information content of a spin  $1/2$  particle was thus

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shown in theory *with certainty* and with *no intermediate observer monitoring/controlling* the process. Many experiments [4-10] have been performed subsequently which provide partial experimental support for this concept. A number of remarkable theoretical concepts and schemes have also been invented for single and multi-particle teleportation [11-18]. The original EPR pair which is maximally entangled and played the role of the carrier of information has been supplemented with less-than-maximally-entangled (with respect to EPR pair) three and four-particle correlated states [19-22].

Some of the difficulties associated with multi-particle teleportation using correlated states are

1. Maximum entanglement in the same sense as in the case of EPR pair cannot be achieved under experimental conditions at present.
2. Three or four-particle correlated states often require additional measurement(s) (Charlie) as an intermediate step as opposed to a pair of particles.
3. The use of three-particle maximally entangled Greenberger-Horne-Zeilinger (GHZ) states for the teleportation of a single particle as quantum carrier and projection basis leads to failure of the process even in theory since four out of eight GHZ states have zero projections for the single-particle leading to null results. This means that it will be impossible for the receiver to reconstruct the unknown state sent by the sender [13].
4. Unitary transformations of the GHZ states which are robust with respect to tracing of one of the particles and which obviate the need for an intermediate observer have not been examined carefully. It must be mentioned that Bell states are merely unitary transformations from other two-particle states, which, however play a vital role in information processing. They are different from an unentangled or partially entangled state by a unitary transformation or two. Therefore it is necessary to identify proper, genuinely entangled states for multiparticle teleportation which has not been done in a systematic manner until now.

5. The extent of correlation between particles which is a direct measure of entanglement is not defined uniformly for multiparticle systems, as against a clear-cut definition for two particle systems [23-25].

In this article we address all of the above problems and propose schemes for both optics experiments and using quantum gates. This article is organized as follows :

- (a) In section II different complete set(s) of orthonormal entangled projection basis of three-particles is (are) proposed by using a unitary transformation of GHZ basis. It is shown that in this new basis the observer Charlie is not needed and direct teleportation results instead of controlled process. Correlation coefficients are proposed as a measurement criterion for entanglement of multiple particles using standard Ursell-Mayer type expansion based on the principles of many body statistical mechanics. In addition to this, three-particle GHZ basis has been used as *quantum carrier* as well as *projection basis* for single particle teleportation.
- (b) In section III two different sets of genuinely entangled four-particle states are proposed for the teleportation of an arbitrary two-particle state. The generalization of the same for  $N$ -particle system is also suggested in detail.
- (c) In section IV teleportation using quantum gates and three and four qubit computational bases is described with the appropriate quantum circuit. This is followed by conclusion.

## II. THREE-PARTICLE ENTANGLED BASIS AND QUANTUM TELEPORTATION WITHOUT PARTICLE AVERAGING

We propose a three-particle basis (123) in the following as

$$\begin{aligned} |\chi\rangle_{123}^{(1),(2)} &= \frac{|\phi\rangle_{12}^+ \otimes |0\rangle_3 \pm |\phi\rangle_{12}^- \otimes |1\rangle_3}{\sqrt{2}} \quad , \quad |\chi\rangle_{123}^{(3),(4)} = \frac{|\phi\rangle_{12}^+ \otimes |1\rangle_3 \pm |\phi\rangle_{12}^- \otimes |0\rangle_3}{\sqrt{2}} \quad , \\ |\chi\rangle_{123}^{(5),(6)} &= \frac{|\psi\rangle_{12}^+ \otimes |0\rangle_3 \pm |\psi\rangle_{12}^- \otimes |1\rangle_3}{\sqrt{2}} \quad \text{and} \quad |\chi\rangle_{123}^{(7),(8)} = \frac{|\psi\rangle_{12}^+ \otimes |1\rangle_3 \pm |\psi\rangle_{12}^- \otimes |0\rangle_3}{\sqrt{2}} \quad (1) \end{aligned}$$

where

$$|\psi\rangle_{12}^\pm = \frac{1}{\sqrt{2}} [ |01\rangle_{12} \pm |10\rangle_{12} ] \quad \text{and} \quad |\phi\rangle_{12}^\pm = \frac{1}{\sqrt{2}} [ |00\rangle_{12} \pm |11\rangle_{12} ] \quad . \quad (2)$$

These states are linear combinations of three-particle GHZ states. The entanglement properties of these states are similar to the three-particle GHZ states if we consider the extent of correlation and to W states for robustness of entanglement with respect to tracing of third particle [31,32]. The extent of correlation of entangled states is measured with the help of the correlation coefficients defined using the well known statistical mechanical formula involving averages for many-body systems [26-30]. Correlation coefficients for two-particle and three-particle systems are defined as

$$C_{\alpha\beta}^{ij} = \langle \sigma_{\alpha}^i \sigma_{\beta}^j \rangle - \langle \sigma_{\alpha}^i \rangle \langle \sigma_{\beta}^j \rangle \text{ and} \quad (3)$$

$$C_{\alpha\beta\gamma}^{ijk} = \langle \sigma_{\alpha}^i \sigma_{\beta}^j \sigma_{\gamma}^k \rangle - \langle \sigma_{\alpha}^i \rangle \langle \sigma_{\beta}^j \sigma_{\gamma}^k \rangle - \langle \sigma_{\beta}^j \rangle \langle \sigma_{\alpha}^i \sigma_{\gamma}^k \rangle - \langle \sigma_{\gamma}^k \rangle \langle \sigma_{\alpha}^i \sigma_{\beta}^j \rangle + 2 \langle \sigma_{\alpha}^i \rangle \langle \sigma_{\beta}^j \rangle \langle \sigma_{\gamma}^k \rangle. \quad (4)$$

where  $\sigma$ 's are the Pauli spin matrices for the indicated particles and  $\{ \alpha, \beta, \gamma = x, y, z \}$ . Table I lists the non-zero correlation coefficients for the four Bell states of a pair of particles. The averages are calculated and compared, for the states  $\{ |\chi\rangle_{123}^{(1)} - |\chi\rangle_{123}^{(8)} \}$  and the eight GHZ states in Table II. The extent of correlation between three particles is the same in both the sets as can be seen from Table II; however, the advantage associated with the entangled basis proposed here over GHZ states is that the states in Eq. 1 are robust with respect to the tracing of 3rd particle, i.e., on tracing the remaining pair is still entangled [31-33]. It is worth mentioning here that a linear combination of three-particle GHZ states such as

$$|\chi\rangle_{123} = \frac{1}{2} [ |000\rangle_{123} + |110\rangle_{123} + |001\rangle_{123} + |111\rangle_{123} ] \quad (5)$$

possesses no genuine three-particle quantum correlation. When calculated, all the correlation coefficients associated with the above state shows zero value, because the state can be expressed as the direct product state  $|\chi\rangle_{123} = \frac{1}{\sqrt{2}} [ |00\rangle_{12} + |11\rangle_{12} ] \otimes \frac{1}{\sqrt{2}} [ |0\rangle_3 + |1\rangle_3 ]$ .

If we use GHZ basis as a quantum channel to teleport unknown information encoded in a single particle then the basis set given by Eq. 1 can be used as projection basis and it obviates the earlier difficulties of missing elements of basis set (when GHZ basis functions are used as a projection basis).

In this scheme, the single-particle information is with Alice, given by  $|\phi\rangle_1 = a|0\rangle_1 + b|1\rangle_1$  where  $\langle\phi_1|\phi_1\rangle = 1$  and  $|a|^2 + |b|^2 = 1$ . The GHZ state (234),  $|\psi\rangle_{234}^{GHZ} = \frac{1}{\sqrt{2}}[|000\rangle_{234} + |111\rangle_{234}]$ , shared by Alice (23) and Bob (4) along with particle (1), gives rise to product representation for the four particle state as

$$|\psi\rangle_{1234} = |\phi\rangle_1 \otimes |\psi\rangle_{234}^{GHZ} = \frac{a[|0000\rangle_{1234} + |0111\rangle_{1234}]}{\sqrt{2}} + \frac{b[|1000\rangle_{1234} + |1111\rangle_{1234}]}{\sqrt{2}}. \quad (6)$$

Alice's measurement process using her basis states  $\{|\chi\rangle_{123}^{(1)} - |\chi\rangle_{123}^{(8)}\}$  is based on the following decomposition of her state,

$$\begin{aligned} |\psi\rangle_{1234} = \frac{1}{2\sqrt{2}} \bigg\{ & |\chi\rangle_{123}^{(1)} [a|0\rangle_4 - b|1\rangle_4] + |\chi\rangle_{123}^{(2)} [a|0\rangle_4 + b|1\rangle_4] + |\chi\rangle_{123}^{(3)} [a|0\rangle_4 + b|1\rangle_4] \\ & + |\chi\rangle_{123}^{(4)} [-a|0\rangle_4 + b|1\rangle_4] + |\chi\rangle_{123}^{(5)} [a|1\rangle_4 + b|0\rangle_4] + |\chi\rangle_{123}^{(6)} [-a|1\rangle_4 + b|0\rangle_4] \\ & + |\chi\rangle_{123}^{(7)} [a|1\rangle_4 - b|0\rangle_4] + |\chi\rangle_{123}^{(8)} [a|1\rangle_4 + b|0\rangle_4] \bigg\}. \end{aligned} \quad (7)$$

Four equally probable results for Bob's particle (4) are possible and also there is no need for another observer to assist Alice in transferring the information to Bob. One of the four probable outcomes is a direct teleportation as indicated by  $I^4$  in Table III. The other three outcomes require one unitary transformation as given in Table III. It is important to mention here that our protocol is an example of *direct teleportation as against controlled teleportation* where Alice needs Charlie to assist her in sending the unknown information to Bob with Charlie carrying one of the entangled particles out of three. In the present case Alice has two particles on her side so that she can do a direct three-particle measurement. Our protocol also overcomes earlier difficulties such as getting null results in four out of eight projections performed by Alice so that the receiver would never be able to recover the unknown state in four such cases.

Gorbachev and Trubilko [13] have shown that, by using a basis which consists of the direct product of a single-particle state with a two-particle entangled state, the teleportation of an arbitrary EPR pair can be realized. In their scheme, Alice interacts her unknown EPR state with the GHZ state and projects her three particles on the three-particle basis states (123), given by  $(|\pi\rangle_1^\pm \otimes |\psi\rangle_{23}^\pm, |\pi\rangle_1^\pm \otimes |\phi\rangle_{23}^\pm)$  where  $|\psi\rangle_{23}^\pm$  and  $|\phi\rangle_{23}^\pm$  are Bell States for the

pair (23) ( Eq. 1) and  $|\pi\rangle_1^\pm = \frac{1}{\sqrt{2}} [ |0\rangle_1 \pm |1\rangle_1 ]$ . This leads to Bob's EPR pair being in a teleported state and the original arbitrary EPR state can be found by doing a simple unitary transformation for all the outcomes. We have demonstrated above that the projection basis in Eq. 1 can be used for satisfactory teleportation of a single particle. To contrast with the result of Gorbachev and Trubilko we give below the scheme to teleport an arbitrary EPR pair through a different set of three-particle entangled projection basis given by

$$\begin{aligned} |\varphi\rangle_{123}^{(1),(2)} &= \frac{|0\rangle_1 \otimes |\phi\rangle_{23}^+ \pm |1\rangle_1 \otimes |\phi\rangle_{23}^-}{\sqrt{2}}, & |\varphi\rangle_{123}^{(3),(4)} &= \frac{|1\rangle_1 \otimes |\phi\rangle_{23}^+ \pm |0\rangle_1 \otimes |\phi\rangle_{23}^-}{\sqrt{2}}, \\ |\varphi\rangle_{123}^{(5),(6)} &= \frac{|0\rangle_1 \otimes |\psi\rangle_{23}^+ \pm |1\rangle_1 \otimes |\psi\rangle_{23}^-}{\sqrt{2}} \text{ and } |\varphi\rangle_{123}^{(7),(8)} &= \frac{|1\rangle_1 \otimes |\psi\rangle_{23}^+ \pm |0\rangle_1 \otimes |\psi\rangle_{23}^-}{\sqrt{2}}. \end{aligned} \quad (8)$$

The sets given by Eq. 1 and Eq. 8 differ in the ordering of the particles. In angular momentum algebraic parlance they refer to different coupling schemes and are related to each other through a unitary transformation containing a 6j coefficient. Alice can use any one of the Bell pairs to transfer information to Bob for e.g.  $|\psi\rangle_{12} = a|01\rangle_{12} + b|10\rangle_{12}$ . The GHZ state  $|\psi\rangle_{345}^{GHZ} = \frac{1}{\sqrt{2}} [ |000\rangle_{345} + |111\rangle_{345} ]$  is composed of Alice's particle 3 and Bob's particles 4 and 5. Thus the unknown two-particle state and the GHZ state give rise to a five-particle state as

$$|\psi\rangle_{12345} = |\psi\rangle_{12} \otimes |\psi\rangle_{345}^{GHZ} = [a|01\rangle_{12} + b|10\rangle_{12}] \otimes \left[ \frac{|000\rangle_{345} + |111\rangle_{345}}{\sqrt{2}} \right]. \quad (9)$$

Alice's measurements are projections on the three-particle (123) states given by Eq. 8, namely,

$$\begin{aligned} |\psi\rangle_{12345} = & \frac{1}{2\sqrt{2}} \left\{ |\varphi\rangle_{123}^{(1)} [a|11\rangle_{45} + b|00\rangle_{45}] + |\varphi\rangle_{123}^{(2)} [a|11\rangle_{45} - b|00\rangle_{45}] + |\varphi\rangle_{123}^{(3)} [-a|11\rangle_{45} + b|00\rangle_{45}] \right. \\ & + |\varphi\rangle_{123}^{(4)} [a|11\rangle_{45} + b|00\rangle_{45}] + |\varphi\rangle_{123}^{(5)} [a|00\rangle_{45} + b|11\rangle_{45}] + |\varphi\rangle_{123}^{(6)} [a|00\rangle_{45} - b|11\rangle_{45}] \\ & \left. + |\varphi\rangle_{123}^{(7)} [-a|00\rangle_{45} + b|11\rangle_{45}] + |\varphi\rangle_{123}^{(8)} [a|00\rangle_{45} + b|11\rangle_{45}] \right\}. \end{aligned} \quad (10)$$

The two-particle (45) state is in one of the four distinct outcomes which can be transformed back to the original arbitrary EPR pair easily through a single qubit unitary transformation by Bob. The required transformations are listed in Table IV.

The basis sets given by Eq. 8 and Eq. 1 work successfully as projection bases for the teleportation of a single particle and an arbitrary EPR pair respectively as well, by doing two different two-qubit transformations on Alice's side. For example, if we consider teleportation of a particle through GHZ state using projection basis given by Eq. 8, the direct product state of four particles is

$$|\psi\rangle_{1234} = \frac{a}{\sqrt{2}} [ |0000\rangle_{1234} + |0111\rangle_{1234} ] + \frac{b}{\sqrt{2}} [ |1000\rangle_{1234} + |1111\rangle_{1234} ]. \quad (11)$$

A direct three-particle measurement using basis set of Eq. 8 will not be able to achieve the desired task, thus Alice does two different unitary transformations on her particles (23) and (12) respectively given by unitary matrices  $U_{23}^{(1)}$  and  $U_{12}^{(2)}$ , where

$$U_{23}^{(1)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}_{23} \quad \text{and} \quad U_{12}^{(2)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}_{12}. \quad (12)$$

The two transformations on  $|\psi\rangle_{1234}$  are represented as

$$|\psi\rangle_{1234}^{(1)} = I_1 \otimes U_{23}^{(1)} \otimes I_4 |\psi\rangle_{1234} = \frac{a}{\sqrt{2}} [ |0000\rangle_{1234} + |0101\rangle_{1234} ] + \frac{b}{\sqrt{2}} [ |1000\rangle_{1234} + |1101\rangle_{1234} ] \quad (13)$$

and

$$|\psi\rangle_{1234}^{(2)} = U_{12}^{(2)} \otimes I_3 \otimes I_4 |\psi\rangle_{1234}^{(1)} = \frac{a}{\sqrt{2}} [ |0000\rangle_{1234} + |0101\rangle_{123} ] + \frac{b}{\sqrt{2}} [ |1100\rangle_{1234} + |1001\rangle_{1234} ] \quad (14)$$

where  $I_i$  represents an identity matrix for  $i^{th}$  particle.  $|\psi\rangle_{1234}^{(2)}$  is re-expressed using the basis set of Eq. 8 for Alice to project her particles onto any of these eight states, as

$$\begin{aligned} |\psi\rangle_{1234}^{(2)} = & \frac{1}{2\sqrt{2}} \left\{ |\varphi\rangle_{123}^{(1)} [a|0\rangle_4 + b|1\rangle_4] + |\varphi\rangle_{123}^{(2)} [a|0\rangle_4 - b|1\rangle_4] + |\varphi\rangle_{123}^{(3)} [a|0\rangle_4 + b|1\rangle_4] \right. \\ & + |\varphi\rangle_{123}^{(4)} [-a|0\rangle_4 + b|1\rangle_4] + |\varphi\rangle_{123}^{(5)} [a|1\rangle_4 - b|0\rangle_4] + |\varphi\rangle_{123}^{(6)} [a|1\rangle_4 + b|0\rangle_4] \\ & \left. + |\varphi\rangle_{123}^{(7)} [-a|1\rangle_4 + b|0\rangle_4] + |\varphi\rangle_{123}^{(8)} [a|1\rangle_4 + b|0\rangle_4] \right\}. \end{aligned} \quad (15)$$

Depending on Alice's measurement Bob may have to apply appropriate unitary transformations to recover Alice's information.

It is worth mentioning here that any of the projection basis given by Eq. 8 can be used as a quantum *carrier* to teleport a single particle using GHZ basis as the *projection basis* for Alice's qubits. The basis set in Eq. 8 possesses same correlation coefficients as that of GHZ states with respect to particle tracing, and is more robust as compared to GHZ states, which makes it a suitable set for being a quantum carrier. The four-particle direct product state composed of Alice's information and the quantum carrier  $|\varphi\rangle_{234}^{(1)}$  is

$$\begin{aligned}
|\psi\rangle_{1234} &= [a|0\rangle_1 + b|1\rangle_1] \otimes |\varphi\rangle_{234}^{(1)} \\
&= \frac{a}{2} [|0000\rangle_{1234} + |0011\rangle_{1234} + |0100\rangle_{1234} - |0111\rangle_{1234}] \\
&= +\frac{b}{2} [|1000\rangle_{1234} + |1011\rangle_{1234} + |1100\rangle_{1234} - |1111\rangle_{1234}]. \tag{16}
\end{aligned}$$

A simple decomposition based on Alice's measurement in the GHZ basis (123) leads to

$$\begin{aligned}
|\psi\rangle_{1234} &= \\
&\frac{1}{2\sqrt{2}} \{ [|000\rangle_{123} + |111\rangle_{123}] [a|0\rangle_4 - b|1\rangle_4] + [|000\rangle_{123} - |111\rangle_{123}] [a|0\rangle_4 + b|1\rangle_4] \\
&+ [|001\rangle_{123} + |110\rangle_{123}] [a|1\rangle_4 + b|0\rangle_4] + [|001\rangle_{123} - |110\rangle_{123}] [a|1\rangle_4 - b|0\rangle_4] \\
&+ [|010\rangle_{123} + |101\rangle_{123}] [a|0\rangle_4 + b|1\rangle_4] + [|010\rangle_{123} - |101\rangle_{123}] [a|0\rangle_4 - b|1\rangle_4] \\
&+ [|011\rangle_{123} + |100\rangle_{123}] [-a|1\rangle_4 + b|0\rangle_4] + [|011\rangle_{123} - |100\rangle_{123}] [-a|1\rangle_4 - b|0\rangle_4] \}. \tag{17}
\end{aligned}$$

The above expression shows direct teleportation of Alice's information in two of the cases with equal probabilities. In all other six cases appropriate unitary transformations are needed subject to Alice's measurement results. In addition to this we would like to mention another but similar set of three-particle states having the same degree of correlation and robustness as states given by Eq. 1 and Eq. 8. These states are used as *projection basis* for the teleportation of a particle and an arbitrary EPR pair through three-particle GHZ state and can be used as a *quantum carrier* for single-particle teleportation using three-particle



GHZ basis as projections. They are given as

$$\begin{aligned} |\chi\rangle_{123}^{(1)',(2)'} &= \frac{|\phi\rangle_{13}^+ \otimes |0\rangle_2 \pm |\phi\rangle_{13}^- \otimes |1\rangle_2}{\sqrt{2}}, & |\chi\rangle_{123}^{(3)',(4)'} &= \frac{|\phi\rangle_{13}^+ \otimes |1\rangle_2 \pm |\phi\rangle_{13}^- \otimes |0\rangle_2}{\sqrt{2}}, \\ |\chi\rangle_{123}^{(5)',(6)'} &= \frac{|\psi\rangle_{13}^+ \otimes |0\rangle_2 \pm |\psi\rangle_{13}^- \otimes |1\rangle_2}{\sqrt{2}} \text{ and } |\chi\rangle_{123}^{(7)',(8)'} &= \frac{|\psi\rangle_{13}^+ \otimes |1\rangle_2 \pm |\psi\rangle_{13}^- \otimes |0\rangle_2}{\sqrt{2}}. \end{aligned} \quad (18)$$

Thus we demonstrate that for the teleportation of a particle, GHZ basis functions can be used as *projections* as well as *quantum carriers*.

### III. MULTIPARTITE ENTANGLEMENT AND TELEPORTATION

In this section the entanglement properties of four- particle entangled states and generalization of the same to multi-particle systems are given. In addition, we discuss the teleportation of arbitrary two-particle state with generalization of the protocol to a  $N$ -particle system using genuine multipartite states as quantum channels. The extent of entanglement is assessed by a well established statistical mechanical formula for correlation coefficients [28-30]. Correlation measures for more than three particles can be defined using Ursell-Mayer type cluster coefficients. The use of quantum virial coefficients [29-30] as a criterion for determining correlations between spins (particles) resolves ambiguities in defining the degree of entanglement between multiple particles as against a clear existing definition for a pair of particles. We therefore, use the following expression for the four particle correlation coefficient,

$$\begin{aligned} C_{\alpha\beta\gamma\delta}^{1234} &= \langle \sigma_\alpha^1 \sigma_\beta^2 \sigma_\gamma^3 \sigma_\delta^4 \rangle - \langle \sigma_\alpha^1 \rangle \langle \sigma_\beta^2 \sigma_\gamma^3 \sigma_\delta^4 \rangle - \langle \sigma_\beta^2 \rangle \langle \sigma_\alpha^1 \sigma_\gamma^3 \sigma_\delta^4 \rangle - \langle \sigma_\gamma^3 \rangle \langle \sigma_\alpha^1 \sigma_\beta^2 \sigma_\delta^4 \rangle \\ &- \langle \sigma_\delta^4 \rangle \langle \sigma_\alpha^1 \sigma_\beta^2 \sigma_\gamma^3 \rangle + 2 \langle \sigma_\alpha^1 \rangle \langle \sigma_\beta^2 \rangle \langle \sigma_\gamma^3 \sigma_\delta^4 \rangle + 2 \langle \sigma_\alpha^1 \rangle \langle \sigma_\gamma^3 \rangle \langle \sigma_\beta^2 \sigma_\delta^4 \rangle + 2 \langle \sigma_\alpha^1 \rangle \langle \sigma_\delta^4 \rangle \langle \sigma_\beta^2 \sigma_\gamma^3 \rangle \\ &+ 2 \langle \sigma_\beta^2 \rangle \langle \sigma_\gamma^3 \rangle \langle \sigma_\alpha^1 \sigma_\delta^4 \rangle + 2 \langle \sigma_\beta^2 \rangle \langle \sigma_\delta^4 \rangle \langle \sigma_\alpha^1 \sigma_\gamma^3 \rangle + 2 \langle \sigma_\gamma^3 \rangle \langle \sigma_\delta^4 \rangle \langle \sigma_\alpha^1 \sigma_\beta^2 \rangle - \langle \sigma_\alpha^1 \sigma_\beta^2 \rangle \langle \sigma_\gamma^3 \sigma_\delta^4 \rangle \\ &- \langle \sigma_\alpha^1 \sigma_\gamma^3 \rangle \langle \sigma_\beta^2 \sigma_\delta^4 \rangle - \langle \sigma_\alpha^1 \sigma_\delta^4 \rangle \langle \sigma_\beta^2 \sigma_\gamma^3 \rangle - 6 \langle \sigma_\alpha^1 \rangle \langle \sigma_\beta^2 \rangle \langle \sigma_\gamma^3 \rangle \langle \sigma_\delta^4 \rangle. \end{aligned} \quad (19)$$

The general expression for the  $N$ -particle correlation coefficient can be obtained by solving the equations for cluster functions derived formally from the  $N$ -th quantum virial coefficient

(quantum trace) [28-29].

Correlation coefficients calculated by using the above expression have been used as entanglement criterion for four-particle states in this paper. Rigolin proposed a set of generalized Bell basis set for the teleportation of two-particle state which is a direct product of two, two-particle Bell states. The four-particle correlation coefficients for all these 16 orthonormal generalized Bell basis are zero. One can form another orthonormal basis set of four-particle states similar to Rigolin's generalized Bell basis, one member of which can be given as

$$|\psi\rangle_{1234}^{(1)} = \frac{1}{2} [ |0000\rangle_{1234} + |1001\rangle_{1234} + |0110\rangle_{1234} + |1111\rangle_{1234} ] \quad (20)$$

This set also works properly for the teleportation of arbitrary two-particle states with only single qubit unitary transformations on Bob's side. However, it also does not have genuine four particle entanglement (all the four-particle correlation coefficients are zero). Rigolin's state(s) and the basis shown above (Eq. 20) are linear combinations of GHZ states and possess no genuine multi-particle correlation which is quite similar to what we have discussed in the previous section with three-particle states (Eq. 5). An example of a set of sixteen four-particle states which possess genuine four-particle correlation (Eq. 19) is given by Yeo and Chua [18]. The non-zero correlation coefficients associated with their set of states is indicated in Table V.

Here we propose two different sets of states which also possess genuine four-particle entanglement which can be used successfully for the teleportation of arbitrary two-particle states.

### A. First set

In the first scheme, we propose a set of orthonormal basis states which are linear combinations of GHZ states, namely

$$\begin{aligned}
|\phi\rangle_{1234}^{(1),(2)} &= \frac{|0\rangle_1 \otimes |\phi\rangle_{23}^+ \otimes |0\rangle_4 \pm |1\rangle_1 \otimes |\phi\rangle_{23}^- \otimes |1\rangle_4}{\sqrt{2}}, \\
|\phi\rangle_{1234}^{(3),(4)} &= \frac{|0\rangle_1 \otimes |\phi\rangle_{23}^+ \otimes |1\rangle_4 \pm |1\rangle_1 \otimes |\phi\rangle_{23}^- \otimes |0\rangle_4}{\sqrt{2}}, \\
|\phi\rangle_{1234}^{(5),(6)} &= \frac{|0\rangle_1 \otimes |\psi\rangle_{23}^+ \otimes |0\rangle_4 \pm |1\rangle_1 \otimes |\psi\rangle_{23}^- \otimes |1\rangle_4}{\sqrt{2}}, \\
|\phi\rangle_{1234}^{(7),(8)} &= \frac{|0\rangle_1 \otimes |\psi\rangle_{23}^+ \otimes |1\rangle_4 \pm |1\rangle_1 \otimes |\psi\rangle_{23}^- \otimes |0\rangle_4}{\sqrt{2}}, \\
|\phi\rangle_{1234}^{(9),(10)} &= \frac{|1\rangle_1 \otimes |\phi\rangle_{23}^+ \otimes |0\rangle_4 \pm |0\rangle_1 \otimes |\phi\rangle_{23}^- \otimes |1\rangle_4}{\sqrt{2}}, \\
|\phi\rangle_{1234}^{(11),(12)} &= \frac{|1\rangle_1 \otimes |\phi\rangle_{23}^+ \otimes |1\rangle_4 \pm |0\rangle_1 \otimes |\phi\rangle_{23}^- \otimes |0\rangle_4}{\sqrt{2}}, \\
|\phi\rangle_{1234}^{(13),(14)} &= \frac{|1\rangle_1 \otimes |\psi\rangle_{23}^+ \otimes |0\rangle_4 \pm |0\rangle_1 \otimes |\psi\rangle_{23}^- \otimes |1\rangle_4}{\sqrt{2}} \quad \text{and} \\
|\phi\rangle_{1234}^{(15),(16)} &= \frac{|1\rangle_1 \otimes |\psi\rangle_{23}^+ \otimes |1\rangle_4 \pm |0\rangle_1 \otimes |\psi\rangle_{23}^- \otimes |0\rangle_4}{\sqrt{2}}. \tag{21}
\end{aligned}$$

The above basis has the advantage that it has genuine four-particle entanglement as these states cannot be written as the direct product of two-particle states. We have calculated the non-zero correlation coefficients for this set and listed them in Table VI. The maximum value ( $\pm 1$ ) of correlation coefficients suggests that the extent of entanglement is maximum between four-particles which supports its use as a quantum carrier as well as a projection basis. These states are robust with respect to two particle tracing (14, 23) such that when traced over (14) or (23), the other two particles will be in a correlated state.

Teleportation protocol for the two particle state  $|\phi\rangle_{12} = a|00\rangle_{12} + b|01\rangle_{12} + c|10\rangle_{12} + d|11\rangle_{12}$  is given below. Alice and Bob can use any one of the entangled state as a quantum carrier described in the given set, e.g.,

$$|\phi\rangle_{3456}^{(1)} = \frac{1}{2} [ |0000\rangle_{3456} + |1001\rangle_{3456} + |0110\rangle_{3456} - |1111\rangle_{3456} ] \tag{22}$$

where particles 3 and 4 are with Alice and 5 and 6 are with Bob. The initial direct product

of the six-particle state is given by

$$\begin{aligned}
|\psi\rangle_{123456} = & \frac{a}{2} [|000000\rangle_{123456} + |001001\rangle_{123456} + |000110\rangle_{123456} - |001111\rangle_{123456}] \\
& + \frac{b}{2} [|010000\rangle_{123456} + |011001\rangle_{123456} + |010110\rangle_{123456} - |011111\rangle_{123456}] \\
& + \frac{c}{2} [|100000\rangle_{123456} + |101001\rangle_{123456} + |100110\rangle_{123456} - |101111\rangle_{123456}] \\
& + \frac{d}{2} [|110000\rangle_{123456} + |111001\rangle_{123456} + |110110\rangle_{123456} - |111111\rangle_{123456}]. \quad (23)
\end{aligned}$$

Direct teleportation is the result, as is seen when Eq. 23 is re-expressed in the basis set of

Eq. 21, i.e.

$$\begin{aligned}
|\psi\rangle_{123456} = & \frac{|\phi\rangle_{1234}^{(1)}}{4} [a|00\rangle_{56} + b|01\rangle_{56} + c|10\rangle_{56} + d|11\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(2)}}{4} [a|00\rangle_{56} + b|01\rangle_{56} - c|10\rangle_{56} - d|11\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(3)}}{4} [a|10\rangle_{56} - b|11\rangle_{56} + c|00\rangle_{56} - d|01\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(4)}}{4} [a|10\rangle_{56} - b|11\rangle_{56} - c|00\rangle_{56} + d|01\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(5)}}{4} [a|01\rangle_{56} + b|00\rangle_{56} - c|11\rangle_{56} - d|10\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(6)}}{4} [a|01\rangle_{56} + b|00\rangle_{56} + c|11\rangle_{56} + d|10\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(7)}}{4} [-a|11\rangle_{56} + b|10\rangle_{56} + c|01\rangle_{56} - d|00\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(8)}}{4} [-a|11\rangle_{56} + b|10\rangle_{56} - c|01\rangle_{56} + d|00\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(9)}}{4} [a|10\rangle_{56} + b|11\rangle_{56} + c|00\rangle_{56} + d|01\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(10)}}{4} [-a|10\rangle_{56} - b|11\rangle_{56} + c|00\rangle_{56} + d|01\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(11)}}{4} [a|00\rangle_{56} - b|01\rangle_{56} + c|10\rangle_{56} - d|11\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(12)}}{4} [-a|00\rangle_{56} + b|01\rangle_{56} + c|10\rangle_{56} - d|11\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(13)}}{4} [-a|11\rangle_{56} - b|10\rangle_{56} + c|01\rangle_{56} + d|00\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(14)}}{4} [a|11\rangle_{56} + b|10\rangle_{56} + c|01\rangle_{56} + d|00\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(15)}}{4} [a|01\rangle_{56} - b|00\rangle_{56} - c|11\rangle_{56} + d|10\rangle_{56}] \\
& + \frac{|\phi\rangle_{1234}^{(16)}}{4} [-a|01\rangle_{56} + b|00\rangle_{56} - c|11\rangle_{56} + d|10\rangle_{56}]. \tag{24}
\end{aligned}$$

Thus if Alice makes a joint measurement on her particles (1234), Bob's two particles (56) will be projected onto one of the sixteen equally probable states. Bob recovers the information by applying appropriate unitary transformations having Alice inform him about her classical outcome(s). The advantage here is the direct product of two single qubit unitary transformation listed in Table VII instead of a joint unitary transformation such as a C-NOT

gate for Bob to recover the unknown information.

### B. Second set

In this subsection we propose another basis which is a linear combination of direct products of the three-particle GHZ states and a single particle as

$$\begin{aligned}
 |\varphi\rangle_{1234}^{(1),(2)} &= \frac{|\chi\rangle_{123}^{(1)''} \otimes |0\rangle_4 \pm |\chi\rangle_{123}^{(3)''} \otimes |1\rangle_4}{\sqrt{2}}, & |\varphi\rangle_{1234}^{(3),(4)} &= \frac{|\chi\rangle_{123}^{(2)''} \otimes |0\rangle_4 \pm |\chi\rangle_{123}^{(4)''} \otimes |1\rangle_4}{\sqrt{2}}, \\
 |\varphi\rangle_{1234}^{(5),(6)} &= \frac{|\chi\rangle_{123}^{(1)''} \otimes |1\rangle_4 \pm |\chi\rangle_{123}^{(3)''} \otimes |0\rangle_4}{\sqrt{2}}, & |\varphi\rangle_{1234}^{(7),(8)} &= \frac{|\chi\rangle_{123}^{(2)''} \otimes |1\rangle_4 \pm |\chi\rangle_{123}^{(4)''} \otimes |0\rangle_4}{\sqrt{2}}, \\
 |\varphi\rangle_{1234}^{(9),(10)} &= \frac{|\chi\rangle_{123}^{(5)''} \otimes |0\rangle_4 \pm |\chi\rangle_{123}^{(7)''} \otimes |1\rangle_4}{\sqrt{2}}, & |\varphi\rangle_{1234}^{(11),(12)} &= \frac{|\chi\rangle_{123}^{(6)''} \otimes |0\rangle_4 \pm |\chi\rangle_{123}^{(8)''} \otimes |1\rangle_4}{\sqrt{2}}, \\
 |\varphi\rangle_{1234}^{(13),(14)} &= \frac{|\chi\rangle_{123}^{(5)''} \otimes |1\rangle_4 \pm |\chi\rangle_{123}^{(7)''} \otimes |0\rangle_4}{\sqrt{2}} & \text{and } |\varphi\rangle_{1234}^{(15),(16)} &= \frac{|\chi\rangle_{123}^{(6)''} \otimes |1\rangle_4 \pm |\chi\rangle_{123}^{(8)''} \otimes |0\rangle_4}{\sqrt{2}}
 \end{aligned} \tag{25}$$

where

$$\begin{aligned}
 |\chi\rangle_{123}^{(1)'', (2)''} &= \frac{1}{\sqrt{2}} [ |000\rangle_{123} \pm |111\rangle_{123} ] \quad , \quad |\chi\rangle_{123}^{(3)'', (4)''} = \frac{1}{\sqrt{2}} [ |010\rangle_{123} \pm |101\rangle_{123} ] , \\
 |\chi\rangle_{123}^{(5)'', (6)''} &= \frac{1}{\sqrt{2}} [ |011\rangle_{123} \pm |100\rangle_{123} ] \quad \text{and} \quad |\chi\rangle_{123}^{(7)'', (8)''} = \frac{1}{\sqrt{2}} [ |001\rangle_{123} \pm |110\rangle_{123} ] \quad (26)
 \end{aligned}$$

are the eight GHZ states corresponding to three particles (123).

The following representation of the above basis set enables us to generate a  $2N$ -particle

entangled basis as described further,

$$\begin{aligned}
|\chi\rangle_{1234}^{(1),(2)} &= \frac{|0\rangle_1 \otimes |\phi\rangle_{24}^+ \otimes |0\rangle_3 \pm |1\rangle_1 \otimes |\psi\rangle_{24}^+ \otimes |1\rangle_3}{\sqrt{2}}, \\
|\chi\rangle_{1234}^{(3),(4)} &= \frac{|0\rangle_1 \otimes |\phi\rangle_{24}^+ \otimes |1\rangle_3 \pm |1\rangle_1 \otimes |\psi\rangle_{24}^+ \otimes |0\rangle_3}{\sqrt{2}}, \\
|\chi\rangle_{1234}^{(5),(6)} &= \frac{|0\rangle_1 \otimes |\phi\rangle_{24}^- \otimes |0\rangle_3 \pm |1\rangle_1 \otimes |\psi\rangle_{24}^- \otimes |1\rangle_3}{\sqrt{2}}, \\
|\chi\rangle_{1234}^{(7),(8)} &= \frac{|0\rangle_1 \otimes |\phi\rangle_{24}^- \otimes |1\rangle_3 \pm |1\rangle_1 \otimes |\psi\rangle_{24}^- \otimes |0\rangle_3}{\sqrt{2}}, \\
|\chi\rangle_{1234}^{(9),(10)} &= \frac{|1\rangle_1 \otimes |\phi\rangle_{24}^+ \otimes |0\rangle_3 \pm |0\rangle_1 \otimes |\psi\rangle_{24}^+ \otimes |1\rangle_3}{\sqrt{2}}, \\
|\chi\rangle_{1234}^{(11),(12)} &= \frac{|1\rangle_1 \otimes |\phi\rangle_{24}^+ \otimes |1\rangle_3 \pm |0\rangle_1 \otimes |\psi\rangle_{24}^+ \otimes |0\rangle_3}{\sqrt{2}}, \\
|\chi\rangle_{1234}^{(13),(14)} &= \frac{|1\rangle_1 \otimes |\phi\rangle_{24}^- \otimes |0\rangle_3 \pm |0\rangle_1 \otimes |\psi\rangle_{24}^- \otimes |1\rangle_3}{\sqrt{2}} \quad \text{and} \\
|\chi\rangle_{1234}^{(15),(16)} &= \frac{|1\rangle_1 \otimes |\phi\rangle_{24}^- \otimes |1\rangle_3 \pm |0\rangle_1 \otimes |\psi\rangle_{24}^- \otimes |0\rangle_3}{\sqrt{2}}.
\end{aligned} \tag{27}$$

These set of states (Eq. 25/Eq. 27) are also maximally entangled four-particle states and cannot be written as direct products of lesser number of particles. Also the four-particle correlation coefficients listed in Table VIII for the above states are non-zero and are also maximal ( $\pm 1$ ). The set has an additional advantage in terms of robustness, i.e. when traced with respect to the 2nd or 4th particle the remaining three particles (123, 134) are entangled. In addition, it is robust with respect to two-particle (13) tracing which leaves other two particle (24) in an entangled state. This makes the above set suitable for the teleportation of an arbitrary two-particle state. Using one of the above states as the state shared by Alice and Bob, namely,

$$|\varphi\rangle_{3456}^{(1)} = \frac{1}{2} [ |0000\rangle_{3456} + |1110\rangle_{3456} + |0101\rangle_{3456} + |1011\rangle_{3456} ] \tag{28}$$

where particles (34) are with Alice and particles (56) are with Bob, the joint state of six-

particles composed of Alice's particles (1234) and Bob's particles (56) give rise to

$$\begin{aligned}
|\psi\rangle_{123456} = & \frac{a}{2} [|000000\rangle_{123456} + |000101\rangle_{123456} + |001110\rangle_{123456} + |001011\rangle_{123456}] \\
& + \frac{b}{2} [|010000\rangle_{123456} + |010101\rangle_{123456} + |011110\rangle_{123456} + |011011\rangle_{123456}] \\
& + \frac{c}{2} [|100000\rangle_{123456} + |100101\rangle_{123456} + |101110\rangle_{123456} + |101011\rangle_{123456}] \\
& + \frac{d}{2} [|110000\rangle_{123456} + |110101\rangle_{123456} + |111110\rangle_{123456} + |111011\rangle_{123456}]. \quad (29)
\end{aligned}$$



Re-expressing Eq. 29 in terms of basis set proposed in (Eq. 25), we have

$$\begin{aligned}
|\psi\rangle_{123456} = & \frac{|\varphi\rangle_{1234}^{(1)}}{4} [a|00\rangle_{56} + b|01\rangle_{56} + c|10\rangle_{56} + d|11\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(2)}}{4} [a|00\rangle_{56} - b|01\rangle_{56} - c|10\rangle_{56} + d|11\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(3)}}{4} [a|00\rangle_{56} + b|01\rangle_{56} - c|10\rangle_{56} - d|11\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(4)}}{4} [a|00\rangle_{56} - b|01\rangle_{56} + c|10\rangle_{56} - d|11\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(5)}}{4} [a|01\rangle_{56} + b|00\rangle_{56} + c|11\rangle_{56} + d|10\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(6)}}{4} [a|01\rangle_{56} - b|00\rangle_{56} - c|11\rangle_{56} + d|10\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(7)}}{4} [a|01\rangle_{56} + b|00\rangle_{56} - c|11\rangle_{56} - d|10\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(8)}}{4} [a|01\rangle_{56} - b|00\rangle_{56} + c|11\rangle_{56} - d|10\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(9)}}{4} [a|10\rangle_{56} + b|11\rangle_{56} + c|00\rangle_{56} + d|01\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(10)}}{4} [-a|10\rangle_{56} + b|11\rangle_{56} + c|00\rangle_{56} - d|01\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(11)}}{4} [a|10\rangle_{56} + b|11\rangle_{56} - c|00\rangle_{56} - d|01\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(12)}}{4} [-a|10\rangle_{56} + b|11\rangle_{56} - c|00\rangle_{56} + d|01\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(13)}}{4} [a|11\rangle_{56} + b|10\rangle_{56} + c|01\rangle_{56} + d|00\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(14)}}{4} [-a|11\rangle_{56} + b|10\rangle_{56} + c|01\rangle_{56} - d|00\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(15)}}{4} [a|11\rangle_{56} + b|10\rangle_{56} - c|01\rangle_{56} - d|00\rangle_{56}] \\
& + \frac{|\varphi\rangle_{1234}^{(16)}}{4} [-a|11\rangle_{56} + b|10\rangle_{56} - c|01\rangle_{56} + d|00\rangle_{56}]. \tag{30}
\end{aligned}$$

It is obvious that, at the most, the unitary transformations which Bob needs to apply reduce to a joint single qubit unitary transformation. Table IX lists all the unitary transformations which might be needed to recover the original state. A different orthogonal set

of states is given by

$$\begin{aligned}
|\varphi\rangle_{1234}^{(1)',(2)'} &= \frac{|\chi\rangle_{123}^{(1)''} \otimes |0\rangle_4 \pm |\chi\rangle_{123}^{(4)''} \otimes |1\rangle_4}{\sqrt{2}}, & |\varphi\rangle_{1234}^{(3)',(4)'} &= \frac{|\chi\rangle_{123}^{(3)''} \otimes |1\rangle_4 \pm |\chi\rangle_{123}^{(2)''} \otimes |0\rangle_4}{\sqrt{2}}, \\
|\varphi\rangle_{1234}^{(5)',(6)'} &= \frac{|\chi\rangle_{123}^{(1)''} \otimes |1\rangle_4 \pm |\chi\rangle_{123}^{(4)''} \otimes |0\rangle_4}{\sqrt{2}}, & |\varphi\rangle_{1234}^{(7)',(8)'} &= \frac{|\chi\rangle_{123}^{(3)''} \otimes |0\rangle_4 \pm |\chi\rangle_{123}^{(2)''} \otimes |1\rangle_4}{\sqrt{2}}, \\
|\varphi\rangle_{1234}^{(9)',(10)'} &= \frac{|\chi\rangle_{123}^{(5)''} \otimes |0\rangle_4 \pm |\chi\rangle_{123}^{(8)''} \otimes |1\rangle_4}{\sqrt{2}}, & |\varphi\rangle_{1234}^{(11)',(12)'} &= \frac{|\chi\rangle_{123}^{(7)''} \otimes |1\rangle_4 \pm |\chi\rangle_{123}^{(6)''} \otimes |0\rangle_4}{\sqrt{2}}, \\
|\varphi\rangle_{1234}^{(13)',(14)'} &= \frac{|\chi\rangle_{123}^{(5)''} \otimes |1\rangle_4 \pm |\chi\rangle_{123}^{(8)''} \otimes |0\rangle_4}{\sqrt{2}} \text{ and } |\varphi\rangle_{1234}^{(15)',(16)'} &= \frac{|\chi\rangle_{123}^{(7)''} \otimes |0\rangle_4 \pm |\chi\rangle_{123}^{(6)''} \otimes |1\rangle_4}{\sqrt{2}}.
\end{aligned} \tag{31}$$

This set also has properties similar to the states given by Eq. 25/Eq. 27. The non-zero four-particle correlation coefficient associated with all the sixteen basis states are listed in Table X. It is an easy exercise to verify that this set works successfully towards teleportation of a two-particle system with only single qubit unitary transformations on Bob's side.

It is possible to generalize the above protocol for the  $N$ -particle system using the basis set given below. The sequential manner in which they are constructed ensures that entanglement properties of these states are preserved down to a pair of particles when they are systematically averaged.

Consider the two-particle Bell states given by Eq. 2 with particles 1 and 2 replaced by 2 and 3. The sixteen four-particle states (1234) can then be given as

$$\frac{1}{\sqrt{2}} \left[ \left( \begin{array}{c} |0\rangle \\ |1\rangle \end{array} \right)_1 \otimes \left( \begin{array}{c} |\phi^+\rangle \\ |\psi^+\rangle \end{array} \right)_{23} \otimes \left( \begin{array}{c} |0\rangle \\ |1\rangle \end{array} \right)_4 \pm \left( \begin{array}{c} |1\rangle \\ |0\rangle \end{array} \right)_1 \otimes \left( \begin{array}{c} |\phi^-\rangle \\ |\psi^-\rangle \end{array} \right)_{23} \otimes \left( \begin{array}{c} |1\rangle \\ |0\rangle \end{array} \right)_4 \right].$$

Relabelling the sixteen states as

$$\begin{pmatrix} |\chi^{(1),(2)}\rangle_{1234} \\ |\chi^{(3),(4)}\rangle_{1234} \\ |\chi^{(5),(6)}\rangle_{1234} \\ |\chi^{(7),(8)}\rangle_{1234} \\ |\chi^{(9),(10)}\rangle_{1234} \\ |\chi^{(11),(12)}\rangle_{1234} \\ |\chi^{(13),(14)}\rangle_{1234} \\ |\chi^{(15),(16)}\rangle_{1234} \end{pmatrix} = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} |0\rangle |\phi^+\rangle |0\rangle \\ |0\rangle |\phi^+\rangle |1\rangle \\ |0\rangle |\psi^+\rangle |0\rangle \\ |0\rangle |\psi^+\rangle |1\rangle \\ |1\rangle |\phi^+\rangle |0\rangle \\ |1\rangle |\phi^+\rangle |1\rangle \\ |1\rangle |\psi^+\rangle |0\rangle \\ |1\rangle |\psi^+\rangle |1\rangle \end{pmatrix} \pm \begin{pmatrix} |1\rangle |\phi^-\rangle |1\rangle \\ |1\rangle |\phi^-\rangle |0\rangle \\ |1\rangle |\psi^-\rangle |1\rangle \\ |1\rangle |\psi^-\rangle |0\rangle \\ |0\rangle |\phi^-\rangle |1\rangle \\ |0\rangle |\phi^-\rangle |0\rangle \\ |0\rangle |\psi^-\rangle |1\rangle \\ |0\rangle |\psi^-\rangle |0\rangle \end{pmatrix} \right], \quad (32)$$

the six-particle generalized entangled states are given by,

$$\frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \otimes \begin{pmatrix} |\chi^{(1)}\rangle_{2345} \\ |\chi^{(3)}\rangle_{2345} \\ |\chi^{(5)}\rangle_{2345} \\ |\chi^{(7)}\rangle_{2345} \\ |\chi^{(9)}\rangle_{2345} \\ |\chi^{(11)}\rangle_{2345} \\ |\chi^{(13)}\rangle_{2345} \\ |\chi^{(15)}\rangle_{2345} \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_6 \pm \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \otimes \begin{pmatrix} |\chi^{(2)}\rangle_{2345} \\ |\chi^{(4)}\rangle_{2345} \\ |\chi^{(6)}\rangle_{2345} \\ |\chi^{(8)}\rangle_{2345} \\ |\chi^{(10)}\rangle_{2345} \\ |\chi^{(12)}\rangle_{2345} \\ |\chi^{(14)}\rangle_{2345} \\ |\chi^{(16)}\rangle_{2345} \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_6 \right].$$

The  $2N$ -particle generalization of the above which contains a set of maximally entangled states can be written down immediately as

$$\begin{aligned} & \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \otimes \begin{pmatrix} |\chi^{(1)}\rangle_{23\dots 2N-1} \\ |\chi^{(3)}\rangle_{23\dots 2N-1} \\ \vdots \\ |\chi^{(2^{2N-2}-3)}\rangle_{23\dots 2N-1} \\ |\chi^{(2^{2N-2}-1)}\rangle_{23\dots 2N-1} \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{2N} \right. \\ & \left. \pm \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \otimes \begin{pmatrix} |\chi^{(2)}\rangle_{23\dots 2N-1} \\ |\chi^{(4)}\rangle_{23\dots 2N-1} \\ \vdots \\ |\chi^{(2^{2N-2}-2)}\rangle_{23\dots 2N-1} \\ |\chi^{(2^{2N-2})}\rangle_{23\dots 2N-1} \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{2N} \right]. \end{aligned}$$

The  $2N$ -particle generalized entangled state for the second set can also be obtained in a similar way as above.

#### IV. TELEPORTATION USING QUANTUM GATES AND COMPUTATIONAL BASIS

In this section we analyze the above schemes using quantum gates and the three and four-qubit computational basis.

##### A. Teleportation of a single qubit through GHZ state

Three qubit states can be prepared by using the appropriate quantum network as given below with the gates required [34-38] on the circuit for the creation of the GHZ state(s). If we give three inputs  $|0\rangle_1$ ,  $|0\rangle_2$  and  $|0\rangle_3$  then the GHZ state  $\frac{1}{\sqrt{2}} [|000\rangle_{123} + |111\rangle_{123}]$  is prepared as indicated in Fig. 1. The quantum circuit required for Alice's unknown qubit to be teleported is shown in Fig. 2. The input for the quantum circuit in Fig. 2 is

$$|\psi\rangle_{1234}^{(0)} = [a|0\rangle_1 + b|1\rangle_1] \otimes \frac{1}{\sqrt{2}} [|000\rangle_{234} + |111\rangle_{234}].$$

Alice sends her qubits 1 and 3 through the C-NOT gate keeping 1 as the control qubit and 3 as the target qubit and obtains

$$|\psi\rangle_{1234}^{(1)} = \frac{a}{\sqrt{2}} [|0000\rangle_{1234} + |0111\rangle_{1234}] + \frac{b}{\sqrt{2}} [|1010\rangle_{1234} + |1101\rangle_{1234}]. \quad (33)$$

Then she sends her qubit 1 through a Hadamard gate with the result

$$\begin{aligned} |\psi\rangle_{1234}^{(2)} &= \frac{a}{2} [|0000\rangle_{1234} + |1000\rangle_{1234} + |0111\rangle_{1234} + |1111\rangle_{1234}] \\ &+ \frac{b}{2} [|0010\rangle_{1234} - |1010\rangle_{1234} + |0101\rangle_{1234} - |1101\rangle_{1234}]. \end{aligned} \quad (34)$$

This she follows by sending qubit 2 through a Hadamard gate again which results in the state as

$$\begin{aligned}
|\psi\rangle_{1234}^{(3)} = & \frac{a}{2\sqrt{2}} [|0000\rangle_{1234} + |0100\rangle_{1234} + |1000\rangle_{1234} + |1100\rangle_{1234} \\
& + |0011\rangle_{1234} - |0111\rangle_{1234} + |1011\rangle_{1234} - |1111\rangle_{1234}] \\
& + \frac{b}{2\sqrt{2}} [|0010\rangle_{1234} + |0110\rangle_{1234} - |1010\rangle_{1234} - |1110\rangle_{1234} \\
& + |0001\rangle_{1234} - |0101\rangle_{1234} - |1001\rangle_{1234} + |1101\rangle_{1234}]. \quad (35)
\end{aligned}$$

A simple rearrangement will decompose the above state into four equally probable measurement outcomes with Bob's particle being projected in one of the four states as

$$\begin{aligned}
|\psi\rangle_{1234}^{(3)} = & \frac{1}{2\sqrt{2}} \{ |000\rangle_{123} [a|0\rangle_4 + b|1\rangle_4] + |001\rangle_{123} [a|1\rangle_4 + b|0\rangle_4] \\
& + |010\rangle_{123} [a|0\rangle_4 - b|1\rangle_4] + |011\rangle_{123} [-a|1\rangle_4 + b|0\rangle_4] \\
& + |110\rangle_{123} [a|0\rangle_4 + b|1\rangle_4] + |101\rangle_{123} [a|1\rangle_4 - b|0\rangle_4] \\
& + |100\rangle_{123} [a|0\rangle_4 - b|1\rangle_4] + |111\rangle_{123} [-a|1\rangle_4 - b|0\rangle_4] \}. \quad (36)
\end{aligned}$$

Table XI lists the required gates on Bob's side which will be activated based on Alice's communication through a classical channel.

### B. Teleportation of an arbitrary EPR pair through GHZ basis

The quantum circuit to accomplish this task is given in Figure 3. In the figure, U's are single qubit unitary transformations on qubits 4 and 5 respectively. Qubits 1, 2 and 3 are with Alice and 4 and 5 are with Bob. The input to the quantum circuit is

$$\begin{aligned}
|\psi\rangle_{12345}^{(0)} = & [a|01\rangle_{12} + b|10\rangle_{12}] \otimes \frac{1}{\sqrt{2}} [|000\rangle_{345} + |111\rangle_{345}] \\
= & \frac{a}{\sqrt{2}} [|01000\rangle_{12345} + |01111\rangle_{12345}] + \frac{b}{\sqrt{2}} [|10000\rangle_{12345} + |10111\rangle_{12345}]. \quad (37)
\end{aligned}$$

The sequence is Alice's transmission of her qubits 1 and 3 through C-NOT gate while keeping qubit 1 as control and 3 as target. This she follows with the transmission of 1 and 2 through

the Hadamard gate. The processes are given in the same sequence by the wave functions

$|\psi\rangle_{12345}^{(1)}$ ,  $|\psi\rangle_{12345}^{(2)}$  and  $|\psi\rangle_{12345}^{(3)}$ , where

$$|\psi\rangle_{12345}^{(1)} = \frac{a}{\sqrt{2}} [|01000\rangle_{12345} + |01111\rangle_{12345}] + \frac{b}{\sqrt{2}} [|10100\rangle_{12345} + |10011\rangle_{12345}], \quad (38)$$

$$\begin{aligned} |\psi\rangle_{12345}^{(2)} &= \frac{a}{2} [|01000\rangle_{12345} + |11000\rangle_{12345} + |01111\rangle_{12345} + |11111\rangle_{12345}] \\ &+ \frac{b}{2} [|00100\rangle_{12345} - |10100\rangle_{12345} + |00011\rangle_{12345} - |10011\rangle_{12345}], \end{aligned} \quad (39)$$

and

$$\begin{aligned} |\psi\rangle_{12345}^{(3)} &= \frac{a}{2\sqrt{2}} [|00000\rangle_{12345} - |01000\rangle_{12345} + |10000\rangle_{12345} - |11000\rangle_{12345} \\ &+ |00111\rangle_{12345} - |01111\rangle_{12345} + |10111\rangle_{12345} - |11111\rangle_{12345}] \\ &+ \frac{b}{2\sqrt{2}} [|00100\rangle_{12345} + |01100\rangle_{12345} - |10100\rangle_{12345} - |11100\rangle_{12345} \\ &+ |00011\rangle_{12345} + |01011\rangle_{12345} - |10011\rangle_{12345} - |11011\rangle_{12345}]. \end{aligned} \quad (40)$$

Decomposing this in terms of the computational three qubit (123) basis set, we get,

$$\begin{aligned} |\psi\rangle_{12345}^{(3)} &= \frac{1}{2\sqrt{2}} \{ |000\rangle_{123} [a|00\rangle_{45} + b|11\rangle_{45}] + |010\rangle_{123} [-a|00\rangle_{45} + b|11\rangle_{45}] \\ &+ |100\rangle_{123} [a|00\rangle_{45} - b|11\rangle_{45}] + |110\rangle_{123} [-a|00\rangle_{45} - b|11\rangle_{45}] \\ &+ |001\rangle_{123} [a|11\rangle_{45} + b|00\rangle_{45}] + |011\rangle_{123} [-a|11\rangle_{45} + b|00\rangle_{45}] \\ &+ |101\rangle_{123} [a|11\rangle_{45} - b|00\rangle_{45}] + |111\rangle_{123} [-a|11\rangle_{45} - b|00\rangle_{45}] \}. \end{aligned} \quad (41)$$

It is clear from the above that Bob's measurements are all equally probable and require at the most one two-qubit gate leading to four equal outcomes. Table XII gives the two qubit gates required for measurements.

### C. Teleportation of a single qubit through entangled basis of three qubits

The three-qubit entangled basis (Eq. 6) can be prepared by applying the Hadamard gate on the first qubit of GHZ basis as

$$\frac{1}{\sqrt{2}} [|000\rangle_{123} + |111\rangle_{123}] \xrightarrow{H^1} \frac{1}{2} [|000\rangle_{123} + |100\rangle_{123} + |011\rangle_{123} - |111\rangle_{123}]$$

The quantum circuit to prepare the above state is given in Fig. 4. The quantum circuit required to teleport the single qubit through the above three qubit entangled state is given in Fig. 5. The input to the circuit is

$$\begin{aligned} |\psi\rangle_{1234}^{(0)} &= \frac{a}{2} [|0000\rangle_{1234} + |0011\rangle_{1234} + |0100\rangle_{1234} - |0111\rangle_{1234}] \\ &+ \frac{b}{2} [|1000\rangle_{1234} + |1011\rangle_{1234} + |1100\rangle_{1234} - |1111\rangle_{1234}]. \end{aligned} \quad (42)$$

The four-qubit direct product state can be decomposed into four equally probable results (similar to Eq. 16 and Eq. 17) on Bob's qubit as

$$\begin{aligned} |\psi\rangle_{1234}^{(3)} &= \frac{1}{2\sqrt{2}} \{ |000\rangle_{123} [a|0\rangle_4 + b|1\rangle_4] + |001\rangle_{123} [a|1\rangle_4 + b|0\rangle_4] \\ &+ |010\rangle_{123} [a|0\rangle_4 - b|1\rangle_4] + |011\rangle_{123} [-a|1\rangle_4 + b|0\rangle_4] \\ &+ |110\rangle_{123} [a|0\rangle_4 + b|1\rangle_4] + |101\rangle_{123} [a|1\rangle_4 - b|0\rangle_4] \\ &+ |100\rangle_{123} [a|0\rangle_4 - b|1\rangle_4] + |111\rangle_{123} [-a|1\rangle_4 - b|0\rangle_4] \}. \end{aligned} \quad (43)$$

The gates required for detection are summarized in Table XI.

#### D. Quantum circuits for two-qubit teleportation

Here we suggest the quantum circuits for preparing different four-qubit entangled states and the network required to teleport arbitrary two qubits through these quantum carriers using four-qubit computational bases.

The quantum circuit required to prepare set of orthonormal states given by Eq. 21 and Eq. 25 are given in figure 6 and figure 8 respectively. Depending on the input given all the 16 orthonormal states can be prepared in the two different sets. In addition to this, figures 7 and 9 provide the quantum network to teleport an arbitrary two-qubit state using the four-particle entangled states (quantum carriers) given by figures 6 and 8, respectively. The symbols have their usual meanings as discussed earlier and algebra related to the process is straightforward. The quantum network to prepare six-qubit entangled state and to teleport

a three qubit arbitrary state through it can be developed on similar grounds. The unitary transformation required on Bob's side are all single qubit unitary transformations and are quite simple to achieve.

## V. CONCLUSION

We have given in this paper schemes for the single-particle teleportation through three-particle GHZ states using three-particle entangled basis set(s) and vice versa. The use of GHZ states and sets of three-particle basis set(s) as *quantum carriers* and as a set of *projection basis* has been explored. Our protocol obviates the earlier difficulties regarding missing basis elements of projection basis and Alice does not need any assistance (Charlie/Cliff) in the process of communication with Bob. In that sense ours is direct teleportation and not a controlled one, in contrast to schemes proposed earlier. We have discussed the entanglement of multipartite states using statistical correlation coefficients and have proposed multiparticle entangled states which possess genuine multiparticle entanglement. We have demonstrated the teleportation of arbitrary two-particle states using the states proposed as quantum carriers with the added advantage of the requirement of only the direct products of single qubit unitary transformations on Bob's side instead of a joint unitary transformation involving two or more particles. We have taken a step forward to suggest the generalization of the protocol for the teleportation of the  $N$ -particle state through a  $2N$ -particle genuinely entangled quantum channel which can be formed by taking proper care to ensure maximum genuine entanglement. In addition, we have analyzed and verified all the protocols discussed here through the use of quantum gates with appropriate quantum circuits.

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Table I

	$ \psi\rangle_{12}^-$	$ \psi\rangle_{12}^+$	$ \phi\rangle_{12}^-$	$ \phi\rangle_{12}^+$
$C_{xx}^{12}$	-1	1	-1	1
$C_{yy}^{12}$	-1	1	1	-1
$C_{zz}^{12}$	-1	-1	1	1

Table II

	$C_{xxx}^{123}$	$C_{yyx}^{123}$	$C_{yxy}^{123}$	$C_{xyy}^{123}$	$C_{xxz}^{123}$	$C_{yyz}^{123}$
$\frac{1}{\sqrt{2}} [  000\rangle_{123} \pm  111\rangle_{123} ]$	$\pm 1$	$\mp 1$	$\mp 1$	$\mp 1$	-	-
$\frac{1}{\sqrt{2}} [  001\rangle_{123} \pm  110\rangle_{123} ]$	$\pm 1$	$\mp 1$	$\pm 1$	$\pm 1$	-	-
$\frac{1}{\sqrt{2}} [  010\rangle_{123} \pm  101\rangle_{123} ]$	$\pm 1$	$\pm 1$	$\mp 1$	$\pm 1$	-	-
$\frac{1}{\sqrt{2}} [  011\rangle_{123} \pm  100\rangle_{123} ]$	$\pm 1$	$\pm 1$	$\pm 1$	$\mp 1$	-	-
$ \chi\rangle_{123}^{(1)}$	-	-	1	1	1	-1
$ \chi\rangle_{123}^{(2)}$	-	-	-1	-1	1	-1
$ \chi\rangle_{123}^{(3)}$	-	-	-1	-1	-1	1
$ \chi\rangle_{123}^{(4)}$	-	-	1	1	-1	1
$ \chi\rangle_{123}^{(5)}$	-	-	1	-1	1	1
$ \chi\rangle_{123}^{(6)}$	-	-	-1	1	1	1
$ \chi\rangle_{123}^{(7)}$	-	-	-1	1	-1	-1
$ \chi\rangle_{123}^{(8)}$	-	-	1	-1	-1	-1

Table III

Measurement outcome	Unitary transformation
$ \chi\rangle_{123}^{(1)}$ , $ \chi\rangle_{123}^{(4)}$	$\sigma_z^4$
$ \chi\rangle_{123}^{(2)}$ , $ \chi\rangle_{123}^{(3)}$	$I^4$
$ \chi\rangle_{123}^{(5)}$ , $ \chi\rangle_{123}^{(8)}$	$\sigma_x^4$
$ \chi\rangle_{123}^{(6)}$ , $ \chi\rangle_{123}^{(7)}$	$\sigma_y^4$



Table IV

Measurement outcome	Unitary transformation
$ \varphi\rangle_{123}^{(1)}$ , $ \varphi\rangle_{123}^{(4)}$	$\sigma_x^4 \otimes I^5$
$ \varphi\rangle_{123}^{(2)}$ , $ \varphi\rangle_{123}^{(3)}$	$\sigma_x^4 \otimes \sigma_z^5$
$ \varphi\rangle_{123}^{(5)}$ , $ \varphi\rangle_{123}^{(8)}$	$I^4 \otimes \sigma_x^5$
$ \varphi\rangle_{123}^{(6)}$ , $ \varphi\rangle_{123}^{(7)}$	$\sigma_z^4 \otimes \sigma_x^5$

Table V

$$|\chi^{00}\rangle_{1234} = \frac{1}{\sqrt{2}} [|\zeta\rangle_{1234}^0 + |\zeta\rangle_{1234}^1] \quad \text{where}$$

$$|\zeta\rangle^0 = \frac{1}{2} [|0000\rangle - |0011\rangle - |0101\rangle + |0110\rangle] \quad \text{and} \quad |\zeta\rangle^1 = \frac{1}{2} [|1001\rangle + |1010\rangle + |1100\rangle + |1111\rangle]$$

	$C_{xyyx}^{1234}$	$C_{xzzx}^{1234}$	$C_{zyyz}^{1234}$	$C_{zzzz}^{1234}$
$ \chi^{00}\rangle_{1234}$	-1	+1	-1	+1
$I^1 \otimes \sigma_x^2 \quad  \chi^{00}\rangle_{1234}$	+1	-1	+1	-1
$I^1 \otimes \sigma_y^2 \quad  \chi^{00}\rangle_{1234}$	-1	-1	-1	-1
$I^1 \otimes \sigma_z^2 \quad  \chi^{00}\rangle_{1234}$	+1	+1	+1	+1
$\sigma_x^1 \otimes I^2 \quad  \chi^{00}\rangle_{1234}$	-1	+1	+1	-1
$\sigma_x^1 \otimes \sigma_x^2 \quad  \chi^{00}\rangle_{1234}$	+1	-1	-1	+1
$\sigma_x^1 \otimes \sigma_y^2 \quad  \chi^{00}\rangle_{1234}$	-1	-1	+1	+1
$\sigma_x^1 \otimes \sigma_z^2 \quad  \chi^{00}\rangle_{1234}$	+1	+1	-1	-1
$\sigma_y^1 \otimes I^2 \quad  \chi^{00}\rangle_{1234}$	+1	-1	+1	-1
$\sigma_y^1 \otimes \sigma_x^2 \quad  \chi^{00}\rangle_{1234}$	-1	+1	-1	+1
$\sigma_y^1 \otimes \sigma_y^2 \quad  \chi^{00}\rangle_{1234}$	+1	+1	+1	+1
$\sigma_y^1 \otimes \sigma_z^2 \quad  \chi^{00}\rangle_{1234}$	-1	-1	-1	-1
$\sigma_z^1 \otimes I^2 \quad  \chi^{00}\rangle_{1234}$	+1	-1	-1	+1
$\sigma_z^1 \otimes \sigma_x^2 \quad  \chi^{00}\rangle_{1234}$	-1	+1	+1	-1
$\sigma_z^1 \otimes \sigma_y^2 \quad  \chi^{00}\rangle_{1234}$	+1	+1	-1	-1
$\sigma_z^1 \otimes \sigma_z^2 \quad  \chi^{00}\rangle_{1234}$	-1	-1	+1	+1

Table VI

	$C_{xxyy}^{1234}$	$C_{xyxy}^{1234}$	$C_{yxyx}^{1234}$	$C_{yyxx}^{1234}$
$ \phi\rangle_{1234}^{(1)}$	+1	+1	+1	+1
$ \phi\rangle_{1234}^{(2)}$	-1	-1	-1	-1
$ \phi\rangle_{1234}^{(3)}$	-1	-1	+1	+1
$ \phi\rangle_{1234}^{(4)}$	+1	+1	-1	-1
$ \phi\rangle_{1234}^{(5)}$	-1	+1	-1	+1
$ \phi\rangle_{1234}^{(6)}$	+1	-1	+1	-1
$ \phi\rangle_{1234}^{(7)}$	+1	-1	-1	+1
$ \phi\rangle_{1234}^{(8)}$	-1	+1	+1	-1
$ \phi\rangle_{1234}^{(9)}$	+1	+1	-1	-1
$ \phi\rangle_{1234}^{(10)}$	-1	-1	+1	+1
$ \phi\rangle_{1234}^{(11)}$	-1	-1	-1	-1
$ \phi\rangle_{1234}^{(12)}$	+1	+1	+1	+1
$ \phi\rangle_{1234}^{(13)}$	-1	+1	+1	-1
$ \phi\rangle_{1234}^{(14)}$	+1	-1	-1	+1
$ \phi\rangle_{1234}^{(15)}$	+1	-1	+1	-1
$ \phi\rangle_{1234}^{(16)}$	-1	+1	-1	+1

Table VII

Measurement outcome	Unitary transformation
$ \phi\rangle_{1234}^{(1)}$	$I^5 \otimes I^6$
$ \phi\rangle_{1234}^{(2)}$	$\sigma_z^5 \otimes I^6$
$ \phi\rangle_{1234}^{(3)}$	$\sigma_x^5 \otimes \sigma_z^6$
$ \phi\rangle_{1234}^{(4)}$	$\sigma_z^5 \otimes \sigma_z^6 \otimes \sigma_x^5$
$ \phi\rangle_{1234}^{(5)}$	$\sigma_z^5 \otimes \sigma_x^6$
$ \phi\rangle_{1234}^{(6)}$	$I^5 \otimes \sigma_x^6$
$ \phi\rangle_{1234}^{(7)}$	$\sigma_z^5 \otimes \sigma_z^6 \otimes \sigma_x^5 \otimes \sigma_x^6$
$ \phi\rangle_{1234}^{(8)}$	$\sigma_x^5 \otimes \sigma_x^6 \otimes \sigma_z^6$
$ \phi\rangle_{1234}^{(9)}$	$\sigma_x^5 \otimes I^6$
$ \phi\rangle_{1234}^{(10)}$	$\sigma_x^5 \otimes \sigma_z^5 \otimes I^6$
$ \phi\rangle_{1234}^{(11)}$	$I^5 \otimes \sigma_z^6$
$ \phi\rangle_{1234}^{(12)}$	$\sigma_z^5 \otimes \sigma_z^6$
$ \phi\rangle_{1234}^{(13)}$	$\sigma_x^5 \otimes \sigma_x^6 \otimes \sigma_z^5$
$ \phi\rangle_{1234}^{(14)}$	$\sigma_x^5 \otimes \sigma_x^6$
$ \phi\rangle_{1234}^{(15)}$	$\sigma_z^5 \otimes \sigma_z^6 \otimes \sigma_x^6$
$ \phi\rangle_{1234}^{(16)}$	$I^5 \otimes \sigma_z^6 \otimes \sigma_x^6$

Table VIII

	$C_{xyz}^{1234}$	$C_{xyy}^{1234}$	$C_{yyx}^{1234}$	$C_{yzy}^{1234}$
$ \varphi\rangle_{1234}^{(1)}$	-1	-1	-1	-1
$ \varphi\rangle_{1234}^{(2)}$	-1	+1	-1	+1
$ \varphi\rangle_{1234}^{(3)}$	+1	+1	+1	+1
$ \varphi\rangle_{1234}^{(4)}$	+1	-1	+1	-1
$ \varphi\rangle_{1234}^{(5)}$	+1	+1	+1	+1
$ \varphi\rangle_{1234}^{(6)}$	+1	-1	+1	-1
$ \varphi\rangle_{1234}^{(7)}$	-1	-1	-1	-1
$ \varphi\rangle_{1234}^{(8)}$	-1	+1	-1	+1
$ \varphi\rangle_{1234}^{(9)}$	-1	-1	+1	+1
$ \varphi\rangle_{1234}^{(10)}$	-1	+1	+1	-1
$ \varphi\rangle_{1234}^{(11)}$	+1	+1	-1	-1
$ \varphi\rangle_{1234}^{(12)}$	+1	-1	-1	+1
$ \varphi\rangle_{1234}^{(13)}$	+1	+1	-1	-1
$ \varphi\rangle_{1234}^{(14)}$	+1	-1	-1	+1
$ \varphi\rangle_{1234}^{(15)}$	-1	-1	+1	+1
$ \varphi\rangle_{1234}^{(16)}$	-1	+1	+1	-1

Table IX

Measurement outcome	Unitary transformation
$ \varphi\rangle_{1234}^{(1)}$	$I^5 \otimes I^6$
$ \varphi\rangle_{1234}^{(2)}$	$\sigma_z^5 \otimes \sigma_z^6$
$ \varphi\rangle_{1234}^{(3)}$	$\sigma_z^5 \otimes I^6$
$ \varphi\rangle_{1234}^{(4)}$	$I^5 \otimes \sigma_z^6$
$ \varphi\rangle_{1234}^{(5)}$	$I^5 \otimes \sigma_x^6$
$ \varphi\rangle_{1234}^{(6)}$	$\sigma_z^5 \otimes \sigma_z^6 \otimes \sigma_x^6$
$ \varphi\rangle_{1234}^{(7)}$	$\sigma_x^6 \otimes \sigma_z^5$
$ \varphi\rangle_{1234}^{(8)}$	$I^5 \otimes \sigma_z^6 \otimes \sigma_x^6$
$ \varphi\rangle_{1234}^{(9)}$	$\sigma_x^5 \otimes I^6$
$ \varphi\rangle_{1234}^{(10)}$	$\sigma_z^6 \otimes \sigma_x^5 \otimes \sigma_z^5$
$ \varphi\rangle_{1234}^{(11)}$	$\sigma_z^5 \otimes \sigma_x^5 \otimes I^6$
$ \varphi\rangle_{1234}^{(12)}$	$\sigma_z^6 \otimes \sigma_x^5$
$ \varphi\rangle_{1234}^{(13)}$	$\sigma_x^5 \otimes \sigma_x^6$
$ \varphi\rangle_{1234}^{(14)}$	$\sigma_x^5 \otimes \sigma_x^6 \otimes \sigma_z^5 \otimes \sigma_z^6$
$ \varphi\rangle_{1234}^{(15)}$	$\sigma_z^5 \otimes \sigma_x^5 \otimes \sigma_x^6$
$ \varphi\rangle_{1234}^{(16)}$	$\sigma_z^6 \otimes \sigma_x^6 \otimes \sigma_x^5$

Table X

	$C_{xxxz}^{1234}$	$C_{zxzx}^{1234}$	$C_{yxyz}^{1234}$	$C_{zyyx}^{1234}$
$ \varphi\rangle_{1234}^{(1)'}_{1234}$	+1	-1	-1	+1
$ \varphi\rangle_{1234}^{(2)'}_{1234}$	+1	+1	-1	-1
$ \varphi\rangle_{1234}^{(3)'}_{1234}$	-1	+1	+1	-1
$ \varphi\rangle_{1234}^{(4)'}_{1234}$	-1	-1	+1	+1
$ \varphi\rangle_{1234}^{(5)'}_{1234}$	-1	-1	+1	+1
$ \varphi\rangle_{1234}^{(6)'}_{1234}$	-1	+1	+1	-1
$ \varphi\rangle_{1234}^{(7)'}_{1234}$	+1	+1	-1	-1
$ \varphi\rangle_{1234}^{(8)'}_{1234}$	+1	-1	-1	+1
$ \varphi\rangle_{1234}^{(9)'}_{1234}$	+1	+1	+1	+1
$ \varphi\rangle_{1234}^{(10)'}_{1234}$	+1	-1	+1	-1
$ \varphi\rangle_{1234}^{(11)'}_{1234}$	-1	-1	-1	-1
$ \varphi\rangle_{1234}^{(12)'}_{1234}$	-1	+1	-1	+1
$ \varphi\rangle_{1234}^{(13)'}_{1234}$	-1	+1	-1	+1
$ \varphi\rangle_{1234}^{(14)'}_{1234}$	-1	-1	-1	-1
$ \varphi\rangle_{1234}^{(15)'}_{1234}$	+1	-1	+1	-1
$ \varphi\rangle_{1234}^{(16)'}_{1234}$	+1	+1	+1	+1

Table XI

Measurement outcome	Unitary transformation
$ 000\rangle_{123}$ and $ 110\rangle_{123}$	$I^4$
$ 001\rangle_{123}$ and $ 111\rangle_{123}$	$X$ gate
$ 010\rangle_{123}$ and $ 100\rangle_{123}$	$Z$ gate
$ 101\rangle_{123}$ and $ 011\rangle_{123}$	$X$ gate followed by $Z$ gate



Table XII

Measurement outcome	Unitary transformation
$ 000\rangle_{123}$ and $ 110\rangle_{123}$	$I^4 \otimes X^5$ gate
$ 001\rangle_{123}$ and $ 111\rangle_{123}$	$X^4$ gate $\otimes I^5$
$ 010\rangle_{123}$ and $ 100\rangle_{123}$	$Z^4$ gate $\otimes X^5$ gate
$ 101\rangle_{123}$ and $ 011\rangle_{123}$	$X^4$ gate $\otimes Z^5$ gate

Note : superscript on a prescribed quantum gates ( $X$  or  $Z$ ) represents the particle index to which the quantum gate must be applied.

Figure 1

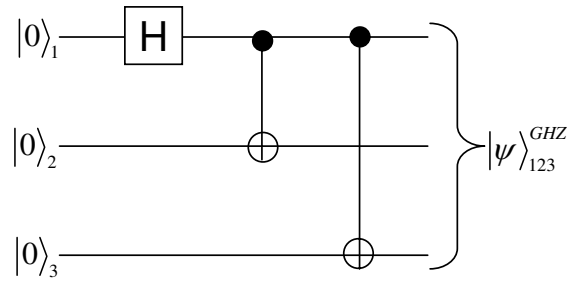


Figure 2

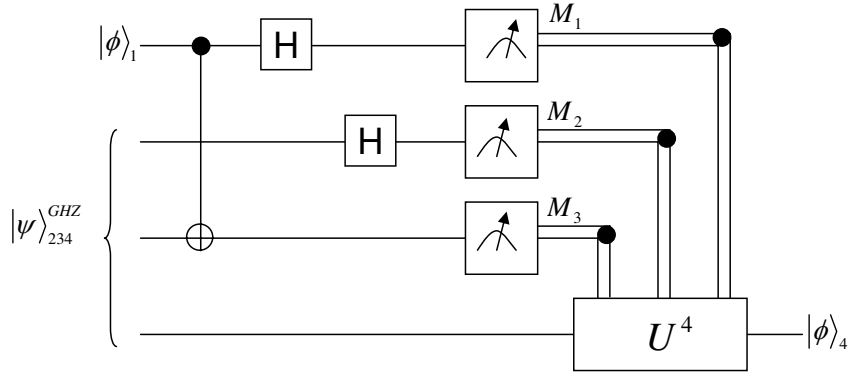


Figure 3

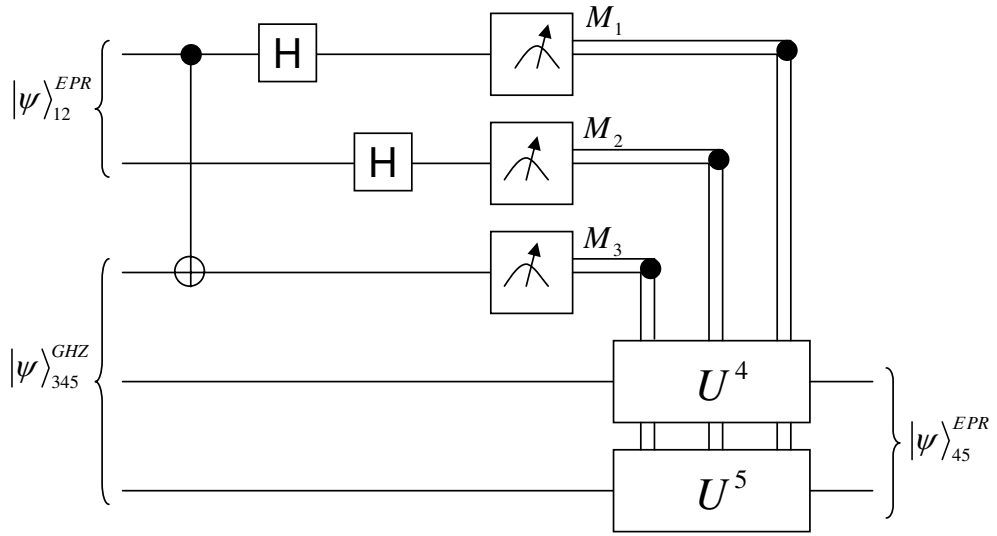


Figure 4

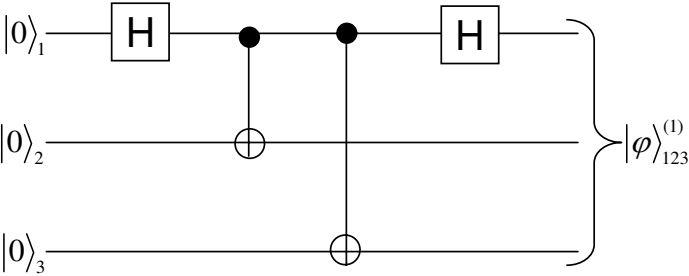


Figure 5

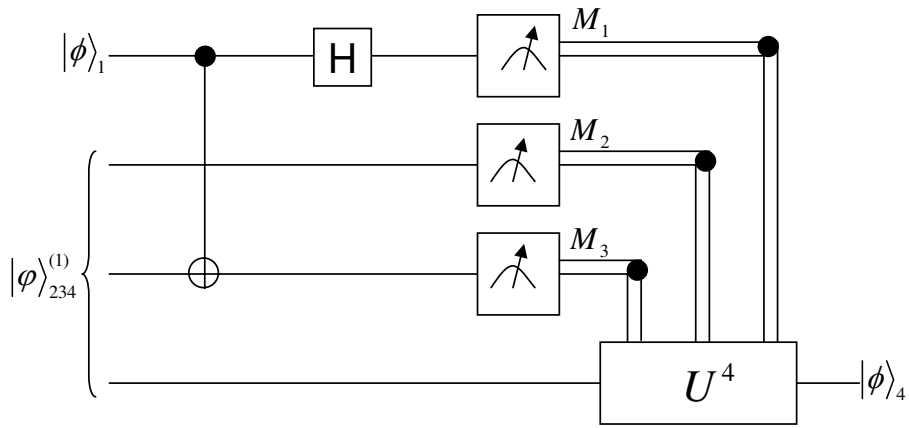


Figure 6

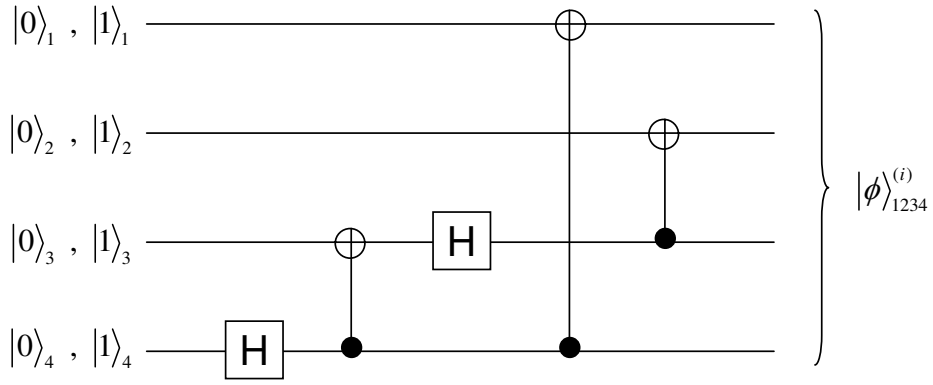


Figure 7

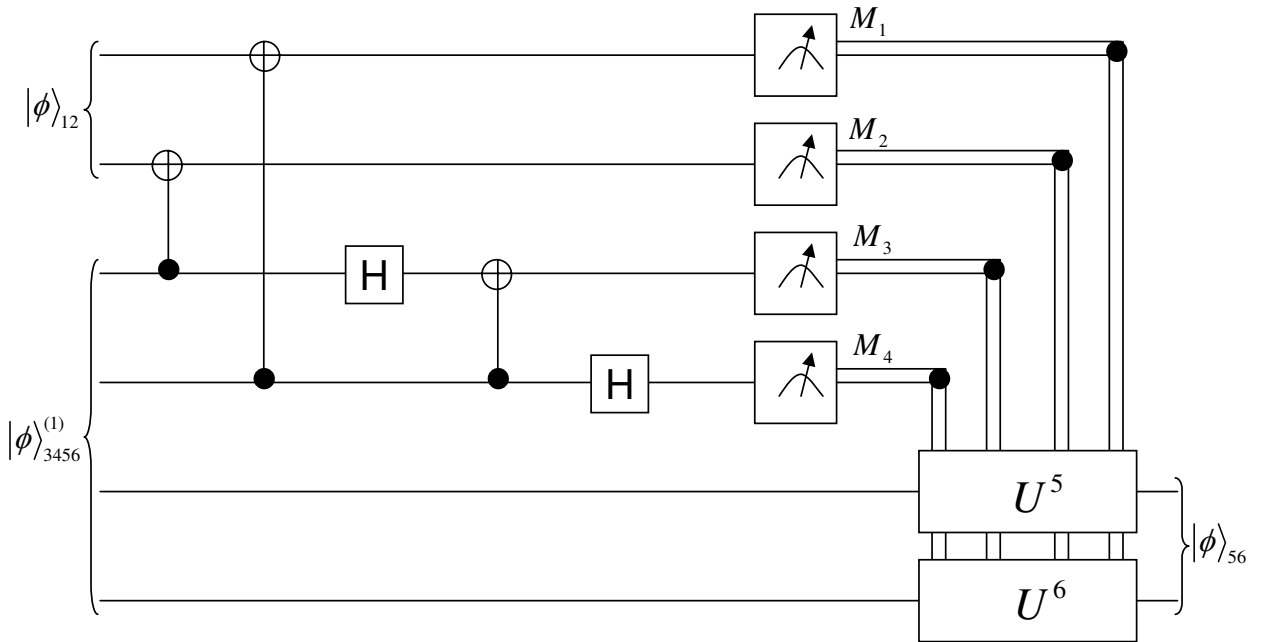




Figure 8

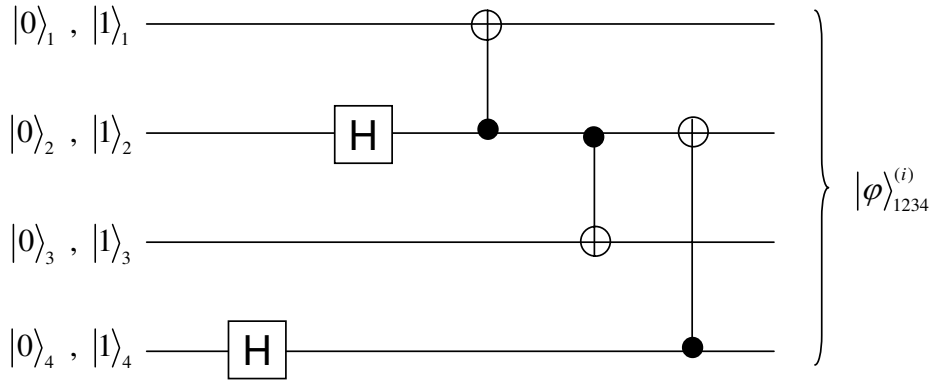


Figure 9

